

# Black holes and traversible wormholes: a synthesis

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## Abstract

A unified framework for black holes and traversible wormholes is described, where both are locally defined by outer trapping horizons, two-way traversible for wormholes and one-way traversible for black or white holes. In a two-dimensional dilaton gravity model, examples are given of: construction of wormholes from black holes; operation of wormholes for transport, including back-reaction; maintenance of an operating wormhole; and collapse of wormholes to black holes. In spherically symmetric Einstein gravity, several exotic matter models supporting wormhole solutions are proposed: ghost scalar fields, exotic fluids and pure ghost radiation.

## 1 Introduction

Space-time wormholes, short cuts between otherwise distant or even unconnected regions of the universe, are now a familiar plot device in science fiction. As a theoretical possibility in General Relativity, they gained some scientific respectability after the article of Morris & Thorne [1]. Apart from attracting public interest and young people to the field, one serious motivation is that wormholes can increase our understanding of gravity when the usual energy conditions are not satisfied, due to quantum effects such as the Casimir effect or Hawking radiation, or in alternative gravitational theories, such as the recently fashionable brane-world models.

## 2 Wormholes and black holes

The author's interest in wormholes originated with the realization that they are very similar to black holes, if one thinks of local properties, rather than the global properties which are usually used to define them. Global traversibility is incompatible with event horizons, leading to the widespread view, even among proponents [1], that wormholes are quite distinct from black holes. However, in terms of local properties, both are characterized by the presence of marginal (marginally trapped) surfaces, and indeed may be defined by outer trapping horizons, types of hypersurface foliated by marginal surfaces [2, 3, 4]. For a static black hole, the event horizons or Killing horizons are examples of outer trapping horizons, and for a static wormhole, the wormhole throat is an example of a double outer trapping horizon, composed of doubly marginal surfaces (Fig.1). The spatial topology of standard black-hole solutions and Morris-Thorne (static, spherically symmetric) wormholes is the same,  $R \times S^2$ , and the spatial geometry can be identical, as for the Schwarzschild black hole and the spatially Schwarzschild wormhole [1]. In each case, a minimal surface connects two asymptotically flat regions.

The key difference is the causal nature of the trapping horizons. This, however, is locally determined by the field equations and so may change with time. Summarizing an earlier proposal [4], black holes and wormholes may be locally defined by outer trapping horizons which are respectively achronal (space-like or null) and temporal (time-like). This means that they are respectively one-way and two-way traversible, as desired in each case. The Einstein equation then shows that they occur respectively under positive and negative energy density, specifically referring to the null energy condition. This means that they are supported respectively by normal matter or vacuum, and what has been dubbed exotic matter [1]. Given the preponderance of normal matter in the universe, this means that they respectively occur naturally and are unlikely to occur naturally, according to present knowledge. However, the possibility of constructing wormholes seems to be open, given the widespread appearance of negative energy densities in quantum

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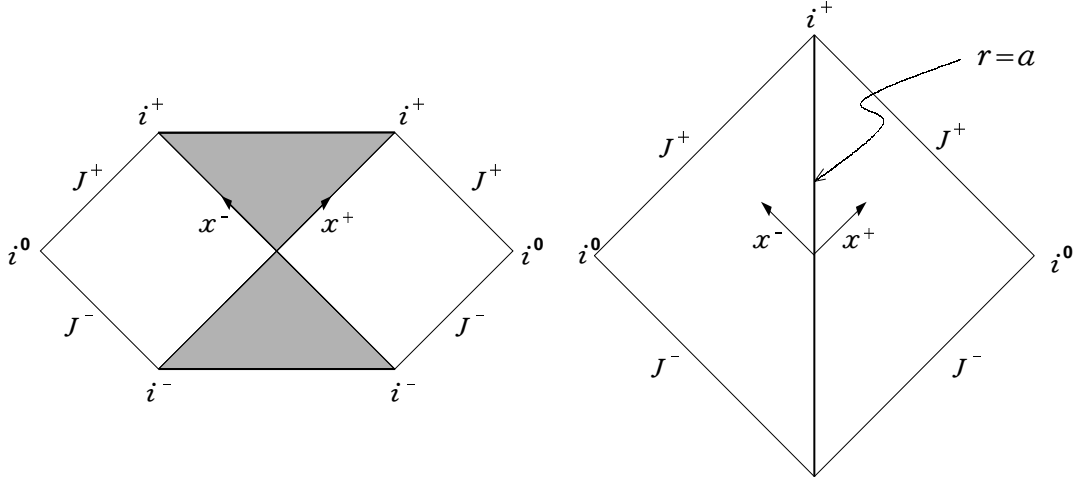


Figure 1: Penrose diagrams of (i) the Schwarzschild black-hole space-time, (ii) a Morris-Thorne wormhole space-time. The black-hole horizons and the wormhole throat are outer trapping horizons, respectively null and time-like.

field theory. Theoretically, if exotic matter can exist in sufficient concentrations, wormholes are as much a prediction of General Relativity as black holes.

A consequence of this synthesis is that black holes and wormholes are interconvertible. The trapping horizons evolve under positive or negative energy density, changing their causal type to locally characterize black holes or wormholes. A wormhole could be converted to a black hole if the supporting exotic matter disperses, or if normal matter is added. Looking at the geometry, the double trapping horizon constituting a static wormhole throat will bifurcate under generic perturbations, forming a trapped region. If the two horizons eventually become null, enclosing a future trapped region, this would be a black hole (Fig.2(i)). Conversely, a black hole could be converted to a wormhole by addition of exotic matter, which causes the two black-hole horizons to become time-like and, if circumstances allow, merge to form a wormhole throat (Fig.2(ii)). In fact, if one considers a Schwarzschild black hole evaporating semi-classically by Hawking radiation, the trapping horizons do indeed become time-like, due to negative-energy radiation absorbed by the black hole, created by pair production with the escaping radiation. The evaporating object is a traversable wormhole by any reasonable definition. This also suggests a wormhole as the endpoint of Hawking evaporation [4, 5].

Finding concrete examples of wormhole and black-hole interconversion requires a specific exotic matter model, so that there are field equations to determine the evolution. Four such models are described below, beginning with a simple model in which analysis is complete, then more realistic models which are under development.

### 3 Two-dimensional dilaton gravity

The CGHS two-dimensional dilaton gravity model [6] was known to contain black-hole solutions analogous to Schwarzschild ones, and to share similar properties such as cosmic censorship [7]. Generalizing the model to include a ghost massless Klein-Gordon field, i.e. with the gravitational coupling taking the opposite sign to normal, leads to static wormhole solutions analogous to Morris-Thorne ones [8]. Moreover, the field equations are explicitly integrable, so it is possible to set initial data corresponding to dynamical perturbations of black holes or wormholes, then analytically find the evolved space-time. Four types of processes have been considered in detail [8], summarized as follows.

**i. Wormhole collapse** to a black hole. Initial data is set so that there is initially a static wormhole, with the supporting ghost radiation then switched off from both sides of the wormhole. The double trapping horizon bifurcates and each section becomes null, forming a black hole. Solutions were found for both sudden and gradual collapse (Fig.2(i)).

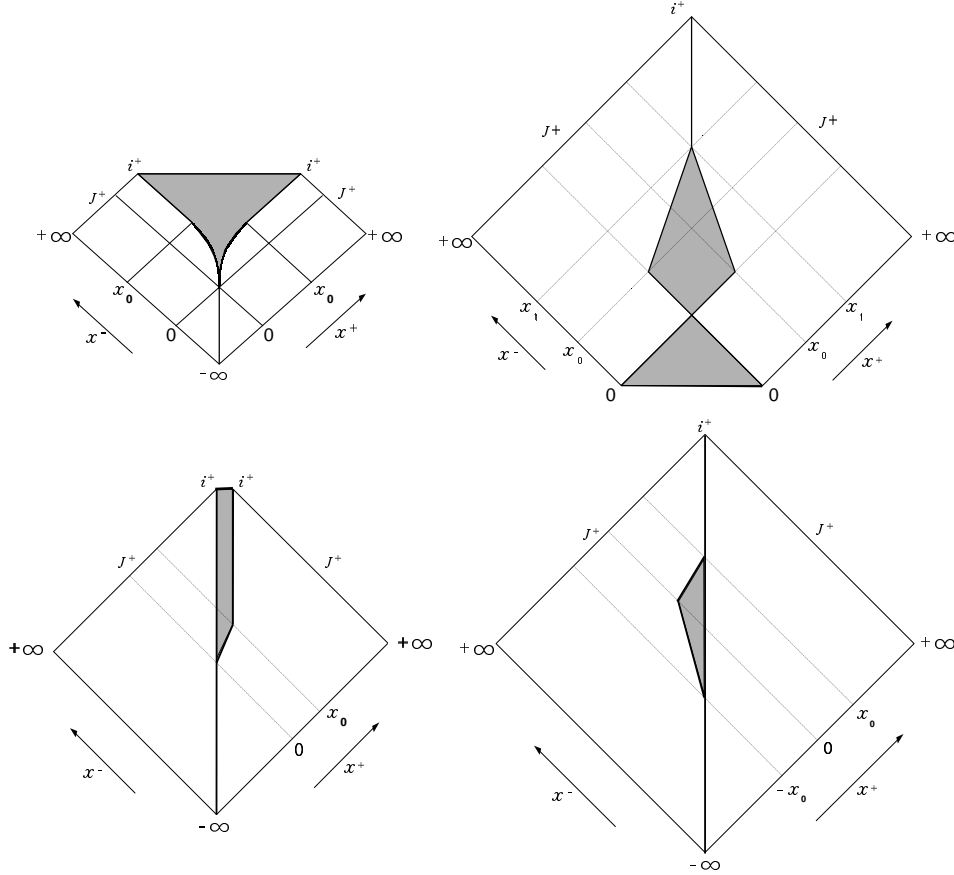


Figure 2: Penrose diagrams of wormhole (i) collapse to a black hole, (ii) construction from a black hole, (iii) operation for transport and (iv) maintenance of a static state, as described in the text. Shading indicates trapped regions. The trapping horizons evolve under positive or negative energy density, changing their causal type accordingly.

**ii. Wormhole construction** from a black hole. Initial data is set so that there is initially a static black hole, which is then irradiated from both sides with the ghost field. The two null trapping horizons of the black hole become time-like and, for appropriate radiation profiles, merge to form the throat of a static wormhole (Fig.2(ii)).

**iii. Wormhole operation** for transport or signalling, including the back-reaction on the wormhole. Initial data is set so that there is initially a static wormhole, with a pulse of normal Klein-Gordon radiation then sent through it. The double trapping horizon again bifurcates, but if the pulse energy is small, the wormhole remains traversible for a long time (Fig.2(iii)). This demonstrates the dynamic stability of the static wormhole.

**iv. Wormhole maintenance** of a static state. As above, but the pulse of normal radiation is preceded by an extra pulse of ghost radiation with equal and opposite energy. The double trapping horizon bifurcates and then merges again, returning the wormhole to its original static state (Fig.2(iv)).

## 4 Exotic matter models

An important shift from the Morris-Thorne approach [1] is to first specify an exotic matter model, then discover (suspiciously often) that wormhole solutions arise naturally. The following exotic matter models are under study in spherically symmetric Einstein gravity.

**i. Ghost scalar fields.** For the massless ghost Klein-Gordon field, there is a classic Morris-Thorne wormhole which was actually found previously by Ellis [9] and several other authors. It has recently been

argued to be stable [10]. A numerical code for spherically symmetric Einstein gravity with a massless ghost Klein-Gordon field has been written and used to study dynamical perturbations of the wormhole [11]. Preliminary results suggest that the Ellis wormhole is unstable, with a weakening ghost field causing collapse to a black hole, and a strengthening ghost field causing an explosion to an inflating universe.

**ii. Exotic fluids**, i.e. fluids not obeying the usual energy conditions. In particular, the spatially Schwarzschild wormhole [1] is a solution for an anisotropic fluid with density zero, radial pressure  $-\tau$  and tangential pressure  $\tau/2$ , where the tension  $\tau$  is positive. This model has vanishing energy trace, like the Maxwell electromagnetic field. Similar models have recently been proposed elsewhere [12].

**iii. Pure ghost radiation**, i.e. pure radiation (or null dust) with negative energy density. With both ingoing and outgoing radiation, static wormhole solutions were conjectured in the conference presentation and found shortly afterwards [13].

## 5 Conclusion

A unified theory of black holes and traversible wormholes has been proposed. Its development includes definitions of mass and surface gravity, and zeroth, first and second laws of black-hole dynamics and wormhole dynamics [2, 3, 4]. This synthesis is useful even for existing problems such as black-hole evaporation, where the Hawking radiation converts a Schwarzschild black hole, at least semi-classically, into a traversible wormhole. It also raises the largely unexplored issue of wormhole thermodynamics.

Black holes are now generally accepted as astrophysical realities, whereas traversible wormholes are often regarded as unphysical theoretical curiosities, outside mainstream scientific research, just as black holes once were. The dual nature of black holes and wormholes, in particular their dynamic interconvertibility, forces new viewpoints. Wormholes are just black holes under negative energy density.

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